AN EVALUATION OF THREE APPROACHES TO TETRAHEDRAL MESH GENERATION FOR DEFORMABLE REGISTRATION OF BRAIN MR IMAGES

Andriy Fedorov, Nikos Chrisochoides[*]

Department of Computer Science
College of William and Mary
Williamsburg, VA 23187, USA

Ron Kikinis, Simon K. Warfield[†]

Surgical Planning Lab
Computational Radiology Lab
Brigham and Women's Hospital
Boston, MA 02115, USA

ABSTRACT

In this paper we evaluate three conceptually different approaches to mesh generation for deformable Finite Element Method (FEM) registration of Magnetic Resonance (MR) images of brain volume. Precise approximation of brain volume segmentations and good shape of the mesh tetrahedra are the main requirements imposed by the application. Our contributions are (1) application-motivated comparison and analysis of practical mesh generation implementations and (2) open source implementation of a mesh generation algorithm which we show delivers mesh quality comparable with the best commercial software products available. The preliminary results indicate, that our implementation provides a solid foundation for further development of application-specific mesh generation tools.

1. INTRODUCTION

Applications which use Finite Element Method (FEM) have first to discretize the modeled object into simple elements, like tetrahedra in 3D. This discretization, or mesh generation, is trivial to accomplish only for simple geometries. The mesh generation problem is complicated even more by the lack of complete understanding of exact requirements to the mesh elements shape [1] and the error distribution over the domain of interest. Requirements for the mesh also differ among FEM applications. Mesh generation has been studied for decades, however, the problem is being continuously addressed in the literature [2–6].

Deformable FEM registration proved to be an effective technique for warping preoperative MRI of different modalities to the intra-operative MR images of low resolution [7, 8]. Unfortunately, we failed to find a practical open source mesh generator for this application. Numerous approaches to mesh generation for medical imaging have been proposed [2, 3, 5, 6]. However, rarely algorithms described in papers are evaluated on realistic datasets and are accompanied by efficient, if any, implementations (notable exceptions include [5, 6, 9]). On the contrary, in this paper we do not propose yet another mesh generation method, but attempt to evaluate existing solutions to the mesh generation and their applicability to FEM registration of brain MRI.

The main contribution of this paper is the evaluation and comparison of representative meshers which implement conceptually different approaches for a specific application: deformable FEM registration of MR images. We also contribute an open implementation of one of the methods, which we show can deliver quality of the discretization comparable with the best commercial software products for our application. The considered mesh generators were evaluated on brain volume segmentations of MR images acquired during past craniotomy cases at Brigham and Women's Hospital [7].

We first discuss motivation and application requirements to mesh generation in Section 2. Section 3 categorizes the most common approaches to tetrahedral mesh generation. The selected representative implementations from each of these categories are presented in Section 4. In Section 5 we outline our evaluation methodology and present evaluation results. We conclude with the summary and directions for future work in Section 6.

2. MOTIVATION AND REQUIREMENTS

Traditionally, mesh generation has been studied for visualization and FEM applications. While for the first area the most important is to deliver efficiently high quality renderings from meshes, FEM make emphasis on the shape of the elements and truthful surface representation. Medical applications of mesh generation include tissue deformation modeling, FEM image registration, surgical planning and visualization, compact shape representation, segmentation. We target FEM registration of preoperative brain volume MRI, fMRI and DT-MRI images with the intra-operative MRI data [7, 8]. The most notable difference from the engineering applications is in the representation of the object of interest. Biological structures and organs usually have smooth surfaces. It is not possible to derive the precise shape information from the imaging data, input may be noisy and topologically incorrect. The FEM model may require smooth mesh on the object outer and, if applicable, internal boundaries (e.g., separating healthy tissue and tumor).

Another set of FEM requirements concerns with the shape of mesh elements. A number of quality metrics have been discussed in [1]. In general, elements with very small and very large dihedral angles should be avoided, as they may affect convergence and accuracy of the solver, or cause high interpolation errors. It is also desirable to have adaptive mesh with smaller elements near the regions of interest (e.g., closer to the object boundary). The requirements imposed on mesh generation by FEM are highly dependent on a particular application [10]. Meshes which are used in FEM registration for this application should be able to tolerate large deformations [7]. Finally, algorithms which can be parallelized [11] are preferred.

ISBI 2006

3. OVERVIEW OF EXISTING APPROACHES

We categorize existing approaches to tetrahedral mesh generation into the following three groups: (1) constrained Delaunay, (2) advancing front, and (3) adaptive space-tree methods. Next we describe in more detail each of the categories together with their strengths and weaknesses.

Delaunay mesh is a triangulation which satisfies the Delaunay criterion (the circumsphere of each edge, face and tetrahedron in 3D is empty). Delaunay triangulations minimize maximum min-containment radius, and the circumradius to shortest edge ratio in the final mesh (thus, maximum interpolation error) can be bounded using Delaunay refinement [1].

Delaunay triangulation of a point set creates a mesh of the convex hull of that set. Boundary of the object has to be recovered from the convex hull using Constrained Delaunay Triangulation (CDT). Existing CDT algorithms have limitations on minimum dihedral angle in the object boundary and often require its explicit definition. It also remains unsolved how to efficiently parallelize Delaunay refinement in 3D [11]. Small angles, which cannot be bounded in Delaunay refinement, may negatively affect the solution quality of iterative solvers [1].

Advancing Front tetrahedralization (AFT) is a mesh generation heuristic which builds mesh starting from the triangulated boundary of the object and moving toward its center [12]. While AFT does not give any guarantees about the final mesh quality, in practice this method has been shown effective.

Quality of AFT meshes depends on the surface mesh quality. It is not trivial how to generate a good quality surface mesh from medical data often defined as a possibly noisy cloud of points. The AFT approach does not introduce any new points on the surface which makes it perfect for parallelization with splitting the original object domain in multiple subdomains. However, there is no robust solution to the subdivision problem: it is difficult to guarantee good geometric properties of subdomains in 3D [11].

Adaptive Space-tree tetrahedrization (AST) approaches are based on a subdivision of the object space into a regular lattice adaptively refined based on user-defined criteria. The main difficulty in all of the AST methods is how to make the mesh conforming to the object boundary [5, 10, 13].

The advantages of AST methods include relative simplicity of the implementation and flexibility in input definition (no explicit surface is required). It is usually possible to parallelize AST approaches with minimum communication between processors.

None of the existing methods in 3D can produce guaranteed quality mesh: Delaunay approaches suffer from *slivers* [1], while other methods are heuristics. In practice, the quality of the final mesh can be improved by applying mesh optimization and sliver exudation techniques [14–17]. However, it has not been proven that quality of the optimized mesh can be bounded.

4. CONSIDERED IMPLEMENTATIONS

In this section we describe some representative implementations from each of the previously defined categories. These implementations were used to evaluate the applicability of different approaches to mesh generation in the context of FEM deformable registration.

TetGen is an open source implementation of Delaunay triangulation and CDT in 3D [9]. *TetMesh-GHS3D* is a commercial mesher supported by Simulog [18]. While the details of the algorithm are not released, the supporting document says that the software "uses a Delaunay-type algorithm". The software was not available to us

at the time of the study, however, we had access to some of the retrospective neurosurgery cases described in [7] where *GHS3D* was used. *SolidMesh* is a commercial package based on AFT [12]. The *SolidMesh* binaries were kindly provided by Dr. David Marcum.

A number of AST methods have been developed recently for medical applications [3–5, 10, 13], but very few implementations are available. We found the crystalline mesh generation method based on body-centric cubic lattice (BCC) [10] the most appropriate for our application. Red green crystalline meshing [10] consists of two phases: (1) construction of the candidate mesh, and (2) compression of the candidate mesh to the object surface (this should not be confused with "data compression"; here we use the original terminology coined in [10]).

The first phase begins with building a regular BCC lattice which encloses the object of interest. This initial lattice is Delaunay and consists of tetrahedra which differ from equilateral tetrahedron as little as possible with regular space tiling [10]. Next, the outside tetrahedra are discarded and the remaining lattice elements are adaptively refined based on the application defined criteria. Red green refinement propagation procedure ensures that the refined mesh is topologically correct. The candidate mesh is selected by heuristically discarding some boundary elements to simplify boundary compression phase. The process of refinement and candidate mesh selection is guided by implicit representation of the object using its distance map. The candidate mesh quality is guaranteed with aspect ratios below 2.1 and dihedral angles between $30°$ and $60°$.

The second phase can be implemented using one of the two approaches. The first approach outfits the candidate mesh with a deformable FEM model [10]. The distance map is then used to set the boundary condition on the surface nodes and push them to the object surface. The second approach is also using the distance map to move the mesh surface nodes to the object surface, followed by element shape optimization.

Following are the observations which motivated our choice:

- the BCC lattice tetrahedra are as close as possible to the equilateral tetrahedron [19], which is much better than the quality of the elements generated by tesselating an adaptive octree, for example, as in [4, 13];

- the mesh boundary compression is independent of the lattice refinement procedure and can be easily substituted with a different implementation;

- the method was designed specifically for modeling large deformations, which is important for registering intra-operative MR images (brain shift of centimeter scale has been observed during some craniotomy procedures [7]);

- lattice-based regular topology and structure of the mesh gives bounds on mesh connectivity and can be advantageous to the FEM application;

- red green refinement can possibly be used to remesh local regions of the mesh without remeshing the whole object;

- lower rate of quality degradation for physics- vs. optimization-based boundary compression has been reported [10].

Unfortunately, the implementation of this method was not available to us. We have developed our own C++ implementation as a collection of new classes within Insight Toolkit (ITK) [20]. It is to the best of our knowledge that this implementation is the first attempt to provide adaptive tetrahedral mesh generation within the Insight Toolkit (the code is available for download as a part of NAMIC SandBox [21]).

5. EVALUATION

The most important requirements imposed on mesh generation by the application are (1) good shape of the tetrahedra (i.e., there are no very small and very large angles) and (2) good surface approximation. The current implementation of the FEM registration does not require intra-operative generation of meshes, thus we rather concentrate on shape and surface approximation evaluation. It is worth mentioning though, that the meshes for our study were generated in less than 10 minutes with most of the time spent in the distance transform computation.

The evaluation was done on three retrospective neurosurgery cases previously used by Clatz et al. [7]. In each case the segmented brain volume had to be meshed. The meshes used in [7] were generated with *GHS3D* software and were all less than 10K elements which imposed additional mesh size constraint on the study. The five sets of meshes we compared were created with

1. red green refinement, physics based FEM boundary compression, linear elastic model (`rgm-p`);

2. red green refinement, optimization based boundary compression using GRUMMP [17] (`rgm-o`);

3. *GHS3D* [18] (`ghs`);

4. CDT *TetGen* v.1.3.4 [9] (`tg`);

5. AFT *SolidMesh* [12] (`sm`).

Methods 4 and 5 from the list above require the surface boundary to be explicitly defined. Our study concentrates on volume mesh generation, thus we make an assumption that the problem of surface reconstruction from medical data is solved and used the surfaces extracted from the volume meshes generated using method 2 as input for methods 4 and 5.

First, we evaluate the quality of surface approximation for each of the methods. We compute the Hausdorff distance from the triangular surfaces reconstructed from segmented images using Marching Cubes algorithm to the surface extracted from volume meshes. We present the results of that evaluation in Table 1. Hausdorff distance was computed using M.E.S.H. software [22]. Note, that we do not evaluate surface approximation for `tg` and `sm` meshes, as their surfaces are identical to those of `rgm-o` meshes.

We evaluate the goodness of element shape using aspect ratio (defined as $\frac{|K|_\infty}{2\sqrt{6}r}$, where $|K|_\infty$ is the length of the longest edge, and r is the radius of the tetrahedron in-circle) and minimum dihedral angle of the tetrahedra. These quality metrics were calculated using VTK 4.4 class `vtkMeshQuality` [23] and are shown in Table 2. Perfect aspect ratio value is 1, and dihedral angles close to $0°$ or $90°$ and larger are considered bad.

Finally, we evaluate the change in quality of the meshes caused by deformation. Aspect ratio and minimum dihedral angles in each of the five meshes after the deformation are shown in Table 3.

The evaluation shows that for all of the cases surface approximation achieved by the mesher we have implemented is at least as good as the approximation achieved by *GHS3D*, which is using Marching cubes followed by decimation to construct the triangular surface mesh used to create the volume mesh [7]. Because of the adaptive mesh structure `rgm` meshes have much more surface elements with about the same number of tetrahedra (see Table 2). This also contributes to a better surface approximation. The element shape quality of the meshes generated with red green method is also comparable and in some cases significantly better than `ghs`, as can be seen from Table 2. CDT meshes generated with *TetGen* have quality comparable with the `ghs` meshes, but they are much large and introduce many small elements near the mesh surface.

Table 1. Surface approximation quality.

case ID	method	surf. faces	Hausdorff distance		
			max	*mean*	*RMS*
1	`rgm-o`	1828	9.16	0.47	0.66
	`rgm-p`	1868	8.73	1.54	1.88
	`ghs`	1284	11.06	0.98	1.25
2	`rgm-o`	2000	6.48	0.55	0.75
	`rgm-p`	2022	7.97	1.40	1.69
	`ghs`	1284	10.37	1.00	1.34
3	`rgm-o`	1858	7.25	0.47	0.63
	`rgm-p`	1896	7.07	1.48	1.81
	`ghs`	1354	8.96	0.87	1.14

Table 2. Element shape quality evaluation.

case ID	method	tets	aspect ratio min/ave/max	min dih. angle min/ave/max
1	`rgm-o`	7334	1.03/1.44/2.83	33.3/52.5/77.2
	`rgm-p`	7565	1.02/1.36/2.56	25.8/55.6/75.6
	`ghs`	7886	1.05/1.61/11.64	6.8/47.8/81.5
	`tg`	21514	1.04/1.97/7.34	11.4/41.7/80.5
	`sm`	8942	1.02/1.37/3.40	17.1/54.0/79.9
2	`rgm-o`	7473	1.02/1.48/4.09	30.7/52.0/79.3
	`rgm-p`	7556	1.02/1.40/2.75	23.6/54.8/76.7
	`ghs`	8202	1.05/1.62/6.68	11.1/47.7/83.3
	`tg`	23907	1.04/1.97/6.24	13.8/41.8/81.8
	`sm`	10266	1.03/1.36/3.45	17.6/54.1/80.6
3	`rgm-o`	7497	1.01/1.46/3.23	32.5/52.1/79.7
	`rgm-p`	7743	1.01/1.37/4.27	14.3/55.4/77.0
	`ghs`	8235	1.02/1.60/19.1	3.07/47.7/83.2
	`tg`	23173	1.04/1.98/6.37	9.0/41.6/82.2
	`sm`	9255	1.03/1.37/3.25	20.5/54.0/79.9

Table 3. Element shape quality evaluation following deformation.

case ID	method	aspect ratio min/ave/max	min dihedral angle min/ave/max
1	`rgm-o`	1.02/1.46/2.80	25.85/52.00/78.79
	`rgm-p`	1.03/1.39/3.36	20.68/54.73/77.03
	`ghs`	1.04/1.65/12.97	5.39/47.33/82.66
	`tg`	1.04/2.06/104.52	0.41/40.92/82.29
	`sm`	1.02/1.40/3.34	17.53/53.44/79.54
2	`rgm-o`	1.02/1.49/4.42	28.28/51.78/77.46
	`rgm-p`	1.02/1.41/2.90	23.40/54.43/78.00
	`ghs`	1.04/1.64/6.34	11.06/47.37/81.41
	`tg`	1.04/2.07/15.60	4.71/40.97/84.61
	`sm`	1.01/1.38/3.54	17.45/53.68/80.33
3	`rgm-o`	1.02/1.49/3.83	17.51/51.46/79.24
	`rgm-p`	1.02/1.41/5.16	13.51/54.26/76.59
	`ghs`	1.05/1.64/22.38	2.43/47.164/83.84
	`tg`	1.05/9.91/171715	0.0004/40.02/83.84
	`sm`	1.02/1.41/3.56	17.93/53.26/80.37

The changes of the mesh quality after deformation is most drastic for Delaunay meshes (see case 3 in Table 3). This is explained by the presence of small tetrahedra near the mesh surface where elements experience largest strains. On average, quality of the elements for all of the methods decreases as the result of deformation. How-

ever, for all of the methods (except `tg`, where we observed large aspect ratios for cases 1 and 3) the deformed meshes preserve the quality and would probably not require remeshing if they were deformed further. We have not observed significant difference in quality change between physics- and optimization-compressed red green meshes.

We emphasize, that `ghs` and `sm` methods are in the commercial domain, and that both `sm` and `tg` require (high quality) surface mesh as input, while `rgm` is open source and operates directly on segmentation data.

6. CONCLUSIONS

Tetrahedral mesh generation has been broadly explored in the past. Most of the approaches have limitations which make them difficult to apply for problems in medical image computing in particular. Boundary fidelity and element shape are the most critical requirements for rapid, robust and accurate registration. We evaluated existing source-available and commercial mesh generators, and our own open source implementation of the red green crystalline mesh generation approach, to assess their capability in this application domain.

Our implementation of the AST approach operates directly on segmentations and generated meshes with boundary fidelity and mesh element quality at least as good (and in some cases significantly better) as of those produced by the state of the art implementations.

In the future we plan to extend the implemented AST method to handle internal object boundaries and evaluate it for different medical applications and other registration approaches which may require local remeshing and tissue removal. One of our immediate goals is to evaluate how the simulation accuracy and convergence is affected by the properties of a simplicial discretization. We believe further research and quantitative mesh generation comparison in this domain will be dramatically facilitated by the ease with which our results may be reproduced as our implementation is available as open source software.

7. ACKNOWLEDGMENTS

8. REFERENCES

[1] J.R. Shewchuk, "What is a good linear finite element? Interpolation, conditioning, and quality measures," in *Proceedings of the 11th IMR*, 2002, pp. 115–126.

[2] A. Mohamed, *Combining Statistical and Biomechanical Models for Estimation of Anatomical Deformations*, Ph.D. thesis, Johns Hopkins University, 2005.

[3] S. Timoner, *Compact Representations for Fast Nonrigid Registration of Medical Images*, Ph.D. thesis, MIT, 2003.

[4] M. Ferrant, A. Nabavi, B.M. Macq, F.A. Jolesz, R. Kikinis, and S.K. Warfield, "Registration of 3d intraoperative MR images of the brain using a finite element biomechanical model," *IEEE Transactions on Medical Imaging*, vol. 20, no. 12, pp. 1384–97, 2001.

[5] G. Berti, "Image-based unstructured 3d mesh generation for medical applications," in *European Congress on Computational Methods in Applied Sciences and Engineering*, 2004.

[6] P.-O. Persson, *Mesh Generation for Implicit Geometries*, Ph.D. thesis, MIT, 2004.

[7] O. Clatz, H. Delingette, I.-F. Talos, A.J. Golby, R. Kikinis, F.A. Jolesz, N. Ayache, and S.K. Warfield, "Robust non-rigid registration to capture brain shift from intraoperative MRI," *IEEE Trans. Med. Imaging*, vol. 24, no. 11, pp. 1417–27, 2005.

[8] N. Archip, A. Fedorov, B. Lloyd, N. Chrisochoides, A. Golby, P.M. Black, and S.K. Warfield, "Integration of patient specific modeling and advanced image processing techniques for image guided neurosurgery," in *Proceedings of SPIE Medical Imaging (to appear)*, 2006.

[9] H. Si and K. Gartner, "Meshing piecewise linear complexes by constrained delaunay tetrahedralizations," in *Proceedings of the 14th IMR*, 2005.

[10] N.P. Molino, R. Bridson, J. Teran, and R. Fedkiw, "A crystalline, red green strategy for meshing highly deformable objects with tetrahedra," in *Proceedings of the 12th IMR*, 2003, pp. 103–114.

[11] N. Chrisochoides, *A Survey of Parallel Mesh Generation Methods*, vol. 51 of *Lecture Notes in Computational Science and Engineering*, Springer Verlag, 2006.

[12] David L. Marcum, "Efficient generation of high-quality unstructured surface and volume grids," *Eng. Comput. (Lond.)*, vol. 17, no. 3, pp. 211–233, 2001.

[13] A. Mohamed and C. Davatzikos, "Finite element mesh generation and remeshing from segmented medical images," in *Proceedings of ISBI'2004*, 2004, pp. 420–423.

[14] P. Alliez, D. Cohen-Steiner, M. Yvinec, and M. Desbrun, "Variational tetrahedral meshing," *ACM Trans. Graph.*, vol. 24, no. 3, pp. 617–625, 2005.

[15] S. Oudot, L. Rineau, and M. Yvinec, "Meshing volumes bounded by smooth surfaces," in *Proceedings of the 14th IMR*, 2005.

[16] L. Freitag and C. Ollivier-Gooch, "A cost/benefit analysis of simplicial mesh improvement techniques as measured by solution efficiency," *International Journal of Computational Geometry*, vol. 10, pp. 361–382, 2000.

[17] GRUMMP, "Generation and Refinement of Unstructured, Mixed-Element Meshes in Parallel," <http://tetra.mech.ubc.ca/GRUMMP/>, 2005, accessed 26 Jan 2006.

[18] Simulog, "GHS3D," <http://www.simulog.fr/mesh/gener2.htm>, 2005, accessed 26 Jan 2006.

[19] A. Fuchs, "Automatic grid generation with almost regular delaunay tetrahedra," in *Proceedings of the 7th IMR*, 1998, pp. 133–147.

[20] A. Fedorov, N. Chrisochoides, R. Kikinis, and S. Warfield, "Mesh generation for medical imaging," in *MICCAI'05 NAMIC Open Source Workshop*, 2005, <http://hdl.handle.net/1926/35>, accessed 26 Jan 2006.

[21] "NAMIC Sandbox repository," <http://svn.na-mic.org:8000/svn/NAMICSandBox/trunk/>, accessed 26 Jan 2006.

[22] M.E.S.H., "Measuring Error of Surface approximation using Hausdorff distance," <http://mesh.berlios.de/>, 2005, accessed 26 Jan 2006.

[23] VTK, "Visualization Toolkit," <http://www.vtk.org/doc/release/5.0/html/a01742.html>, 2005, accessed 27 Jan 2006.